

EXISTENCE REGION, TRUE BULK PHASE CONCENTRATION, AND  
HYDRAULIC RESISTANCE FOR AN ANNULAR FLOW OF A GAS-LIQUID  
MIXTURE IN A TUBE

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Generalized relationships are derived for the basic characteristics of a two-phase annular flow.

It is possible to base reliable methods of calculating two-phase flows on generalized relationships for the major characteristics, which can be used to close the system of equations describing the motion.

The flow structure affects the dependence of the true bulk concentrations, the hydraulic-resistance coefficients, and the heat-transfer ones on the definitive criteria for a two-phase flow, so it is important to be able to determine the flow structure. In studies dealing with calculation methods, a change in flow structure is related not to change in the visually observable boundary between the phases but to change in the true volume concentrations of the phases and the hydraulic-resistance coefficient as affected by the flow-rate and other parameters.

We have determined the changes in flow structure from a change in the dependence of the true volume concentration on the criteria as this is a major parameter defining the hydraulic and thermal conditions in a pipeline. The essence of the method is to derive the joint solution between two equations describing the relationships for the concentrations with different structures.

To use laboratory results to calculate a real system, the relationships must be constructed in dimensionless form. Here a major task is to identify the definitive similarity criteria for each structure. If there is a plug structure, the true gas content is closely described by the Froude number and by the representation of the experimental data in the form  $\varphi = \varphi(\beta; Fr)$  has become traditional [1-4]. The definitive criteria for an annular flow have repeatedly been discussed, but at present there is no agreed view on the subject.

The usual treatment is based on the true concentration as a function of flow rate and the modified Froude number [2, 3], although measurements on vertical tubes of various diameters [4] have shown that the definitive criterion for the annular structure should be one that does not contain the tube diameter. This condition is met by the product of the basic similarity criteria for a two-phase flow: the Reynolds and Froude numbers. The dimensionless combination  $Re_1 Fr$  is suggested as the definitive parameter for an annular structure.

This suggestion has been checked by experimental examination of the true liquid volume content for given values of  $Re_1 Fr$  ( $Re_1 = u_c D / \nu_1$ ;  $Fr = u_c^2 / gD$ ). The gas-liquid flow took place in a horizontal glass tube of diameter 15.2 mm and length 5.8 m at atmospheric pressure. The liquid was either water or a solution of M20 oil in diesel fuel. The viscosity was adjusted via the concentration. The true volume content of liquid was measured by cutting off the experimental section from the pipeline. The length of the cutoff section was 2.8 m. The experiments were performed with  $(Re_1 Fr)^{1/3} = 140, 187, 280$ , and the viscosity of the liquid varied from  $1 \times 10^{-3}$  to  $22.1 \times 10^{-3}$  N·sec/m<sup>2</sup>, while the speeds varied from 3 to 17 m/sec.

Figure 1 shows the results. The experimental points for various speeds and viscosities group satisfactorily along the  $(Re_1 Fr)^{1/3} = \text{const}$  lines in the range of flow rates where the interface between the phases is only slightly perturbed. Therefore, this product can be used in describing the experimental data on true liquid contents for annular flow structures.

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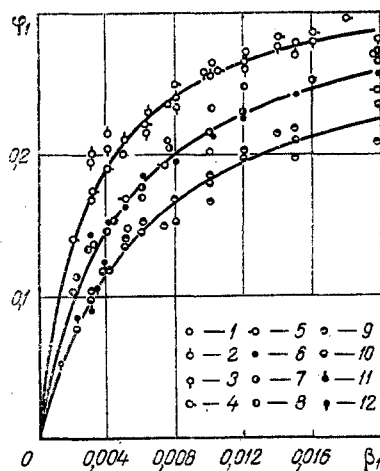


Fig. 1. Dependence of the true liquid content on the flow-rate value and the parameter  $(Re_1 Fr)^{1/3}$ :  $(Re_1 Fr)^{1/3} = 140$ : 1)  $\mu_1 = 1 \cdot 10^{-3}$  N: sec/m<sup>2</sup>; 2)  $7.8 \cdot 10^{-3}$ ; 3)  $13.1 \cdot 10^{-3}$ ; 4)  $22.1 \cdot 10^{-3}$ ;  $(Re_1 Fr)^{1/3} = 187$ : 5)  $\mu_1 = 1 \cdot 10^{-3}$ ; 6)  $7.8 \cdot 10^{-3}$ ; 7)  $13.1 \cdot 10^{-3}$ ; 8)  $22.1 \cdot 10^{-3}$ ;  $(Re_1 Fr)^{1/3} = 240$ : 9)  $\mu_1 = 1/10^{-3}$ ; 10)  $7.8 \cdot 10^{-3}$ ; 11)  $13.1 \cdot 10^{-3}$ ; 12)  $22.1 \cdot 10^{-3}$ .

To determine the dependence of the true liquid content on the tube orientation, we performed experiments with air bubbling through the liquid in glass tubes with  $D = 15.2$  and  $32.7$  mm with angles of inclination to the horizontal  $\alpha = 1^\circ 30'$ ,  $3^\circ$ ,  $6^\circ$ ,  $10^\circ$ ,  $25^\circ$  and  $90^\circ$ ; Figure 2a shows the results for  $D = 15.2$  mm.

In the speed range  $u_c < 3.5-4$  m/sec, one gets plug flow. There was no effect of the tube orientation on true liquid content. At speeds  $u_c > 3.5-4$  m/sec, there was a clear-cut boundary between the liquid and the gas. The liquid was unsymmetrically distributed with respect to the axis (apart from the case  $\alpha = 90^\circ$ ). Much of it lay along the lower generator, while the upper generator remained unwetted. Flow asymmetry persisted up to complete removal of the liquid from the tube.

The following is the empirical formula for the true liquid content under these circumstances attained by processing our results together with the data of [5], where the experiments were performed with vertical tubes with variable reduced phase densities  $\bar{\rho} = \rho_2/\rho_1$  from  $1.36 \times 10^{-3}$  to  $25 \times 10^{-3}$ , and also the data of [4] (vertical tubes, reduced viscosities  $\bar{\mu} = \mu_2/\mu_1$  varying from  $1.02 \times 10^{-3}$  to  $1.27 \times 10^{-3}$ ):

$$\varphi_{1b} = 0.0053 \frac{3.3 - W}{V_1^{1/3}} \quad \text{for } W < 3.3, \quad (1)$$

$$\varphi_{1b} = 0 \quad \text{for } W \geq 3.3,$$

where

$$W = u_c \left( \frac{\rho_1 - \rho_2}{\sigma g \sin \alpha} \right)^{0.25} \left( \frac{\rho_2}{\rho_1} \right)^{0.5}; \quad V_1 = \left( Re_1 Fr \frac{\rho_2}{\rho_1 - \rho_2} \right)^{1/3}$$

Experiments were performed also with an air-water flow in a tube with  $D = 15.2$  mm with  $u_c = 4, 6, 8, 10, 12,$  and  $14$  m/sec. The motion of mixture was ascending ( $\alpha = 90^\circ, 16^\circ, 6^\circ$ ), horizontal ( $\alpha = 0^\circ$ ), or descending ( $\alpha = -6^\circ, -16^\circ, -90^\circ$ ). The curves (Fig. 2b and c) show that the tube orientation plays a substantial part at low speeds. The effect diminishes as the speed rises, which is due to the increasing importance of inertial forces relative to gravitational ones. An increase in liquid content reduces the effects of the angle of inclination on  $\varphi_1$ .

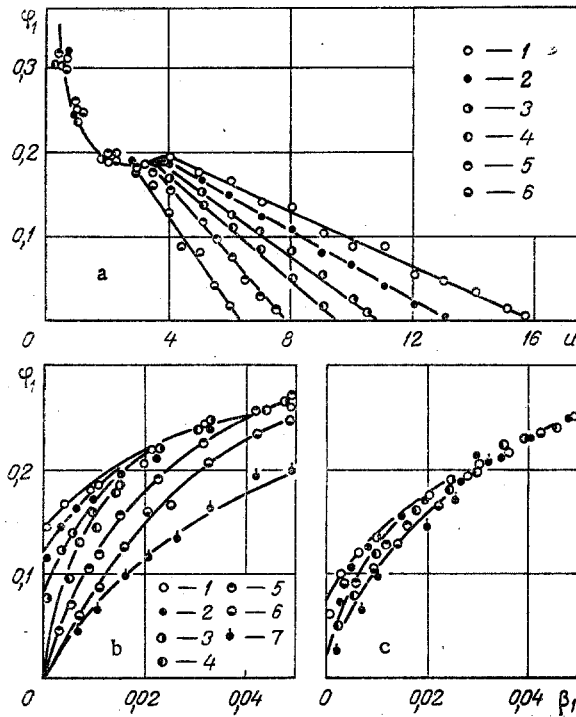


Fig. 2. True liquid content for an air-water mixture in tubes with various orientations: a) bubbling; 1)  $\alpha = 90^\circ$ ; 2)  $25^\circ$ ; 3)  $10^\circ$ ; 4)  $6^\circ$ ; 5)  $3^\circ$ ; 6)  $1^\circ 30'$ ; b)  $u_c = 6$  m/sec; c)  $u_c = 12$  m/sec: 1)  $\alpha = 90^\circ$ ; 2)  $16^\circ$ ; 3)  $6^\circ$ ; 4)  $0^\circ$ ; 5)  $-6^\circ$ ; 6)  $-16^\circ$ ; 7)  $-90^\circ$ .

For rising motion, the  $\varphi_1 = \varphi_1(\beta_1; u_c; \alpha)$  curves make intercepts on the ordinate at  $\varphi_1 = \varphi_{1b}$ , which can be found from (1). An empirical formula was derived for the true liquid content in rising motion:

$$\varphi_1 = \varphi_{1b}(1 + 200\beta_1)^{-1} + 5.5V_1^{-1} \sqrt{100\beta_1}. \quad (2)$$

In descending motion, an increase in the angle of inclination reduced the true liquid content. All the  $\varphi_1 = \varphi_1(\beta_1; u_c; \alpha)$  curves for  $\beta_1 = 0$  converge on the point  $\varphi_1 = 0$ . The following is the empirical formula for the true liquid content in horizontal or descending motion of an annular flow:

$$\varphi_1 = 5.5[1 - |\sin \alpha|^{1.66} (1 + 3.86 \cdot 10^{-6} V_1^3)^{-1}] V_1^{-1} \sqrt{100\beta_1}. \quad (3)$$

The boundary between the annular structure and the rod one is determined from the start of deviation from a linear relationship as characteristic of the rod structure [4]. The measurements showed that the tube orientation did not influence the position of the boundary (see Fig. 2a for example). An increase in speed caused the ring-structure zone to extend to higher values of  $\beta_1$ .

We examined the data of [6] derived from steam-water flows in vertical tubes at pressures of 3.5 and 7 MPa to determine the effects of pressure on the transition boundary. The effects of the liquid viscosity were evaluated from the data of [4].

The following is the empirical equation for the boundary between the annular and rod structures for a two-phase mixture obtained by processing our data and those of [4, 6-9]:

$$V = V^*, \quad (4)$$

where

$$V = \left( \text{ReFr} \frac{\rho_2}{\rho_1 - \rho_2} \right)^{1/3}; \quad V^* = (8.2 - 0.0017 \bar{\mu}^{-0.6}) \exp[(8 + 62\bar{\mu}) \beta_1];$$

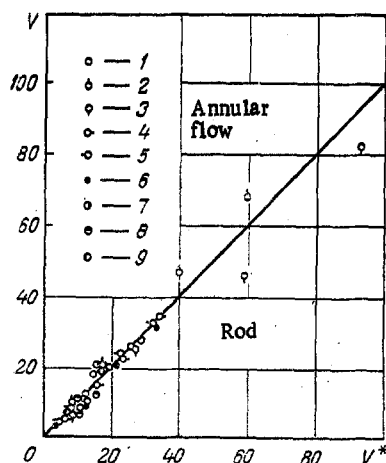


Fig. 3. Comparison of the theoretical relationship of (4) with the experimental data: 1) our data; data of [6] (steam-water mixtures, vertical tube with  $D = 10.2$  mm); 2)  $p = 3.5$  MPa; 3)  $p = 7$  MPa; data of [4] (air-oil mixtures, vertical tubes with  $D = 15.2$  and  $32.7$  mm): 4)  $\bar{\mu} = 1.02 \cdot 10^{-3}$ ; 5)  $\bar{\mu} = 4.04 \cdot 10^{-4}$ ; 6)  $\bar{\mu} = 1.27 \cdot 10^{-3}$ ; 7) data of [7]; 8) [8]; 9) [9].

$$Re = u_c D / \nu_c; \quad 1/\nu_c = \beta_1/\nu_1 + \beta_2/\nu_2.$$

One gets a rod structure when the left side of (4) is larger than the right and an annular one when the converse applies.

Figure 3 compares calculations from (4) with the available experimental data.

There are two groups of methods for calculating the hydraulic resistance for an annular flow of a gas-liquid mixture. The first includes the use of separated models, in which one examines the motion of each phase, while the conditions at the interface are defined by means of semiempirical theories [3, 10, 11]. It is difficult to set up engineering calculation methods on the basis of these models because we lack an adequate volume of experimental evidence on the local characteristics of annular flows and phenomena at the phase boundary.

In practical applications, the main use is made of methods based on homogeneous models, which use various methods of averaging the flow and physical characteristics over the cross section and with respect to time [2, 12]. The essence of this is to define the relationship between the resistance coefficient and the defining criteria.

To determine the hydraulic-resistance coefficient, we use a one-dimensional equation of motion:

$$\frac{dp}{dx} = \lambda_c \left( \frac{\beta_1^2}{\varphi_1} \rho_1 + \frac{\beta_2^2}{\varphi_2} \rho_2 \right) \frac{u_c^2}{2D}. \quad (5)$$

The parametric dependence for  $\lambda_c$  is solved in the form [13]

$$\lambda_c = \lambda_0(Re; \varepsilon) \psi, \quad (6)$$

where  $\lambda_0(Re; \varepsilon)$  is the resistance coefficient for the flow of a homogeneous medium, which is dependent on the Reynolds number and on the relative roughness.

The experimental data on the reduced hydraulic-resistance coefficient for tubes of various diameters are closely described by means of the Froude number [4] in the case of a rod structure, and empirical formulas for  $\psi$  are constructed in the form

$$\psi = \psi(\beta; \bar{\rho}; Fr). \quad (7)$$

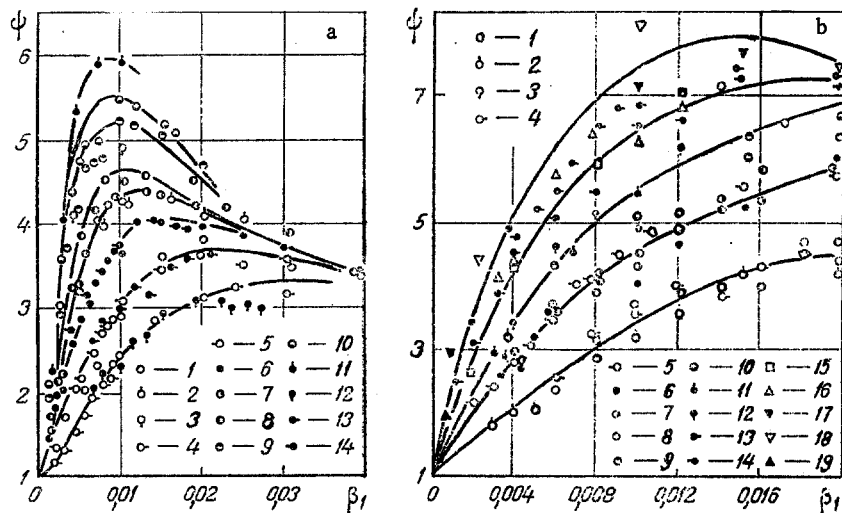


Fig. 4. Dependence of the reduced hydraulic-resistance coefficient for a gas-liquid flow on the flow-rate liquid content and mixture speed for tubes of various diameters (a) and for various viscosities of the liquid phase (b): a)  $D = 9.1$  mm: 1)  $u_c = 10$  m/sec; 2) 14; 3) 20;  $D = 15.2$  mm: 4)  $u_c = 8$  m/sec; 5) 10; 6) 12; 7) 14; 8) 16; 9) 20; 10) 25; 11) 30;  $D = 32.7$  mm: 12)  $u_c = 8$  m/sec; 13) 10; 14) 12; b)  $u_c = 6$  m/sec: 1)  $\mu_1 = 1 \cdot 10^{-3}$  N·sec/m<sup>2</sup>; 2)  $7.8 \cdot 10^{-3}$ ; 3)  $11.3 \cdot 10^{-3}$ ; 4)  $19 \cdot 10^{-3}$ ;  $u_c = 8$  m/sec; 5)  $\mu_1 = 1 \cdot 10^{-3}$  N·sec/m<sup>2</sup>; 6)  $7.8 \cdot 10^{-3}$ ; 7)  $11.3 \cdot 10^{-3}$ ; 8)  $19 \cdot 10^{-3}$ ;  $u_c = 10$  m/sec; 9)  $\mu_1 = 1 \cdot 10^{-3}$  N·sec/m<sup>2</sup>; 10)  $16.3 \cdot 10^{-3}$ ; 11)  $93 \cdot 10^{-3}$ ; 12)  $104 \cdot 10^{-3}$ ;  $u_c = 12$  m/sec; 13)  $\mu_1 = 1 \cdot 10^{-3}$  N·sec/m<sup>2</sup>; 14)  $7.8 \cdot 10^{-3}$ ; 15)  $11.3 \cdot 10^{-3}$ ; 16)  $19 \cdot 10^{-3}$ ;  $u_c = 15$  m/sec; 17)  $\mu_1 = 1 \cdot 10^{-3}$  N·sec/m<sup>2</sup>; 18)  $16.3 \cdot 10^{-3}$ ; 19)  $93 \cdot 10^{-3}$ .

To define the criteria for the reduced coefficients for an annular flow, we performed three series of experiments with gas-liquid mixtures in horizontal tubes. The first series involved in air-water mixture in glass tubes of internal diameters  $D = 9.1, 15.2,$  and  $32.7$  mm at atmospheric pressure. The measurements were made with speeds of  $u_c = 8, 10, 12, 14, 16, 20, 25,$  and  $30$  m/sec. The pressure difference was measured with a U differential manometer. There were separating vessels to prevent the liquid from entering the pulse lines. The true volume concentrations required in (5) were measured by the cutoff method. Measurements with air were used to determine  $\lambda_0 = \lambda_0(\text{Re}; \epsilon)$  for the tubes.

Figure 4a shows the results processed from (5) and (6). The Reynolds number for the mixture was defined as

$$\text{Re} = u_c D (\beta_1 / \nu_1 + \beta_2 / \nu_2). \quad (8)$$

No matter what the tube diameter, the points lie along a line of constant mixture speed, which indicates that a criterion not containing the diameter should be the definitive one for the reduced coefficient with an annular structure.

To establish the detailed form of the relationship for the resistance coefficient, we made measurements with gas-liquid mixtures having various viscosities (from  $10^{-3}$  to  $104 \times 10^{-3}$  N·sec/m<sup>2</sup>).

In the second series of experiments, the measurements were made at fixed values of  $\text{Re}_1, \text{Fr}$ ; the homogeneous-liquid resistance coefficient was derived from the definition of the Reynolds number in (8). The results separated in accordance with the viscosity of the liquid.

The third series of experiments was performed with given velocities  $u_c = 6, 8, 10, 12,$  and  $15$  m/sec. Here  $\lambda_0$  was calculated for

$$\text{Re} = u_c D / \nu_1. \quad (9)$$

Figure 4b shows the results from this series. No matter what the viscosity of the liquid phase, the points fall along lines corresponding to constant velocities.

We use the data of [11] to incorporate the effects of the component densities on  $\psi$ .

The following is the empirical formula for the reduced resistance coefficient with an annular structure for a gas-liquid mixture:

$$\psi = 1 + 0.0033 \left( \text{ReFr} \frac{\rho_1 - \rho_2}{\rho_2} \right)^{1/3} \exp[-15(\bar{\rho} + \beta_1)] \sqrt{100\beta_1}. \quad (10)$$

The value of  $\lambda_0$  was determined for the value of the Reynolds number given by (9).

#### NOTATION

$u$ , viscosity, m/sec;  $p$ , pressure, Pa;  $D$ , internal tube diameter, m;  $\rho$ , density, kg/m<sup>3</sup>;  $\mu$ , dynamic viscosity, N·sec/m<sup>2</sup>;  $\nu$ , kinematic viscosity, m<sup>2</sup>/sec;  $\sigma$ , surface tension, N/m;  $\beta$ , volumetric flow rate concentration, dimensionless;  $\varphi$ , true volumetric concentration, dimensionless;  $g$ , acceleration of gravity, m/sec<sup>2</sup>;  $\lambda$ , hydraulic resistance coefficient, dimensionless;  $\psi$ , reduced hydraulic resistance coefficient, dimensionless;  $\epsilon$ , relative roughness, dimensionless;  $x$ , coordinate along the axis parallel to the horizontal plane, m;  $\alpha$ , angle of inclination to the horizontal plane, deg. Numbers: Reynolds, Re; Froude, Fr. Subscripts: 1, liquid; 2, gas; c, mixture; b, bubbling.

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